

Water absorption and measurement of the mass diffusivity in porous media

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Abstract—The study on water absorption in porous media is briefly reviewed. The mean mass diffusivity for moisture migration in the sense of a constant property is related to the actual mass diffusivity, and its dependence on the initial moisture content is presented. A method is proposed to determine the parameters in the exponential expression of the mass diffusivity according to the mean mass diffusivity measured. The corresponding error transmission is discussed, and experimental results from wet sand are compared with the predicted curves.

1. INTRODUCTION

MORE ATTENTION has been paid to the study on heat and mass transfer in wet porous media in recent years. In order to establish, verify and apply these theories it is necessary to determine the heat and moisture transport properties in wet porous media and to ascertain their dependence upon the temperature and moisture content of the media. The mass diffusivity in porous media, for example, depends strongly upon their moisture content and may vary typically through three orders of magnitude from nearly saturated to nearly dry conditions. Consequently, the constant property assumption, which is usually accepted in a heat transfer study, is inadequate in solving moisture transport problems in general. The capillary hysteresis, the heterogeneity in the structure and porosity and the possible swelling or shrinking of the medium matrix on wetting or drying further aggravate difficulties in the property measurements. As a result, there are few data available in the literature.

The majority of the measurements of heat and moisture transport properties in wet porous media have been designed so far on the basis of the constant property assumption [1, 2] due to the difficulty in solving the non-linear differential equation governing these transport phenomena. Thus it is inevitable to find out the relationship between the actual property and that determined on the assumed constant property in specific testing conditions. The analysis of isothermal moisture migration in porous media has been most intensively developed in soil science with a moisture content dependent diffusivity concerned [3, 4]. Some of the techniques developed in soil science are drawn on in our research. This paper tries to relate the mean mass diffusivity determined on the constant property assumption with the actual one from the study of a fundamental process of isothermal absorption in porous media.

2. SOLUTION OF THE NON-LINEAR DIFFUSION EQUATION

Supposing the gravitational effect is negligible, the isothermal moisture migration in a homogeneous porous medium is described by the non-linear diffusion equation [1, 2]

$$\frac{\partial w}{\partial \tau} = \nabla \cdot (D_m \nabla w) \quad (1)$$

where D_m denotes the mass diffusivity of the medium, which will be a function of the moisture content, w , only.

The isothermal absorption process studied here is the moisture movement in a one-dimensional semi-infinite medium with uniform initial moisture content w_i and a boundary being kept at a constant moisture content w_0 since a certain instant. Thus the process can be formulated as

$$\frac{\partial w}{\partial \tau} = \frac{\partial}{\partial x} \left(D_m \frac{\partial w}{\partial x} \right) \quad (2)$$

with initial and boundary conditions

$$x > 0 \quad \text{and} \quad \tau = 0, \quad w = w_i \quad (3)$$

$$x = 0 \quad \text{and} \quad \tau > 0, \quad w = w_0. \quad (4)$$

Such a problem has been analysed in detail in refs. [3, 4].

By the application of Boltzman's transformation, i.e. introducing

$$\eta = x\tau^{-1/2} \quad (5)$$

equation (2) can be simplified to the following ordinary differential equation:

$$\frac{d}{d\eta} \left(D_m \frac{dw}{d\eta} \right) + \frac{1}{2} \eta \frac{dw}{d\eta} = 0 \quad (6)$$

with the conditions

NOMENCLATURE

c	coefficient in the exponential expression of diffusivity, equation (12)	W	normalized moisture content
C	$c(w_o - w_i)$	x	coordinate [m].
D_m	mass diffusivity [$m^2 s^{-1}$]	Greek symbols	
$D_{m,o}$	coefficient in the exponential expression of diffusivity [$m^2 s^{-1}$]	η	$x\tau^{1/2}$
\bar{D}_m	mean mass diffusivity [$m^2 s^{-1}$]	ρ_d	bulk density of the dry medium [$kg m^{-3}$]
F	function defined in equation (24)	τ	time [s]
h	length of the sections [m]	ϕ	normalized initial moisture content.
I	cumulative absorption [$kg m^{-2}$]	Subscripts	
M	confluent geometric function	i	initial
R	D_m/D_m^*	o	on the boundary
w	moisture content [kg (moisture)/kg (dry medium)]	$*$	reference case.

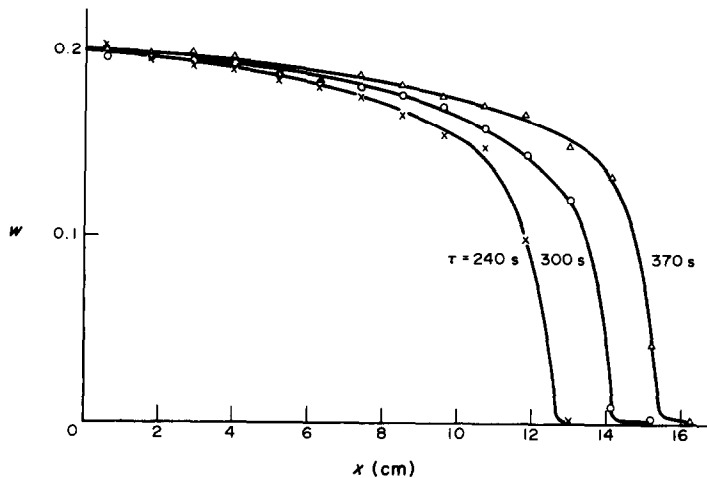


FIG. 1. Moisture content profiles measured in a sand specimen on isothermal absorption.

$$\eta = 0, \quad w = w_o \quad (7)$$

$$\eta \rightarrow \infty, \quad w = w_i. \quad (8)$$

The solution of this problem has the form

$$x(w, \tau) = \eta(w)\tau^{1/2}. \quad (9)$$

Equation (9) predicts that the penetrating moisture content profile advances with a rate proportional to the square root of time. Such an advance is seen in Fig. 1, where the moisture profiles were determined in our experiments on sand. When replotted against $x\tau^{-1/2}$, these profiles fall approximately on a single $w-\eta$ master curve as shown in Fig. 2.

It can also be derived from equation (9) that the cumulative absorption, i.e. the total amount of water taken into the medium by absorption, is

$$I(\tau) = \rho_d \tau^{1/2} \int_{w_i}^{w_o} \eta \, dw \quad (10)$$

which varies simply with $\tau^{1/2}$ as well.

Except for a few special mathematical cases, the

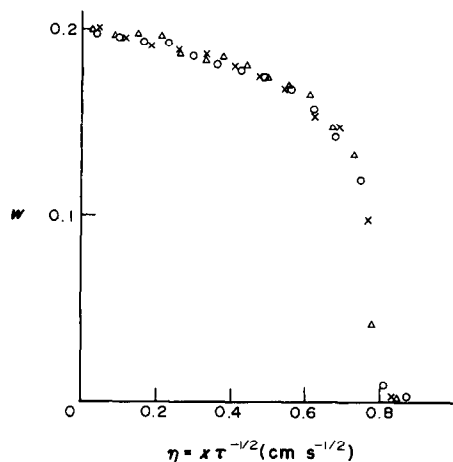


FIG. 2. $w-\eta$ master curve of the sand specimen.

exact solution of the non-linear diffusion equation cannot be expressed in a concise explicit formulation. In equation (9) $\eta(w)$ may be calculated from diffu-

sivity data $D_m(w)$ by numerical procedures, especially that due to Philip [5]. However, several approximate solutions of the absorption problem, i.e. equation (6), have been proposed [6, 7], and Brutsaert's solution [7] appears to be the most accurate one available in the literature. It takes the form

$$\eta = \left[\frac{1}{2} \int_0^1 W^{1/2} D_m(W) dW \right]^{-1/2} \int_w W^{-1/2} D_m(W) dW \quad (11)$$

where $W = (w - w_i)/(w_o - w_i)$ is the normalized moisture content.

As mass diffusivity of porous media, such as soil and building materials, is strongly dependent on their moisture content, an exponential expression was first proposed by Gardner and Mayhugh [8] and later confirmed by many others to describe approximately the moisture content dependence of the mass diffusivity. The expression is

$$D_m = D_{m,o} \exp [c(w - w_o)] \quad (12)$$

where $D_{m,o}$ is the mass diffusivity at a reference moisture content w_o and c is an empirical constant for a specific medium.

3. MEAN MASS DIFFUSIVITY IN THE SENSE OF CONSTANT PROPERTY

In the linearized model, the mass diffusivity of the media is assumed constant, i.e.

$$D_m(w) = \text{const.} = \bar{D}_m \quad (13)$$

Then the exact solution of equation (2) subject to conditions (3) and (4) will be

$$\frac{w - w_i}{w_o - w_i} = \text{erfc} (x/2\sqrt{(\bar{D}_m \tau)}). \quad (14)$$

As a result, the cumulative water absorption of the medium can be derived from equation (10) as

$$I(\tau) = 2\rho_d(w_o - w_i) (\bar{D}_m \tau/\pi)^{1/2} \quad (15)$$

or

$$\bar{D}_m = \frac{\pi}{4\tau} \left[\frac{I}{\rho_d(w_o - w_i)} \right]^2 \quad (16)$$

The cumulative absorption I may be measured gravimetrically or with a burette described below, and then \bar{D}_m is determined from equation (16). This method makes it easier to obtain results with reasonable accuracy. However, such a mean mass diffusivity does not make any general sense in moisture transport calculations except the isothermal absorption process discussed here. Thus our purpose is to look for the relation between this mean mass diffusivity and the actual one.

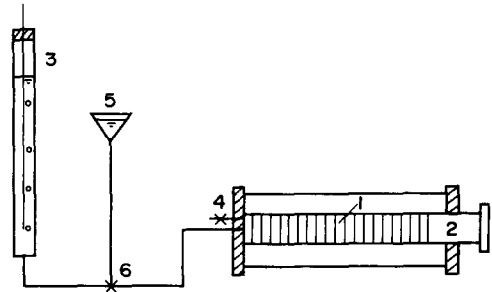


FIG. 3. Apparatus for determining moisture distribution in absorption.

4. METHOD FOR DETERMINING $D_m(w)$

A method was first proposed by Bruce and Klute [9] to determine the mass diffusivity as a function of moisture content. Integrating equation (6), the mass diffusivity can be expressed as

$$D_m = \frac{1}{2} \frac{d\eta}{dw} \int_{w_i}^w \eta dw. \quad (17)$$

Thus, on measuring experimentally the moisture content profile in a specimen for isothermal absorption at an instant τ or the history of the moisture content variation at a given cross-section, a curve $w = w(\eta)$ like that shown in Fig. 2 can be obtained, and accordingly the mass diffusivity is determined by equation (17). While employing this method, Bruce and Klute obtained the moisture content profile by the traditional gravimetric procedure of cutting and weighing. Later on, the non-destructive nuclear magnetic resonance (NMR) imaging technique was reported for detecting both the distribution and history of moisture content in specimens [10]. This application of NMR imaging was a prominent advance in the study of moisture migration in porous media, nevertheless the high cost of equipment has hampered its application in this field.

Although Bruce and Klute's method has the merit of determining a whole function of $D_m(w)$ in a single test, it suffers from the difficulty in determining with precision the derivative $d\eta/dw$, which requires a smooth experimental curve. The observed wetting profile is, however, inevitably distorted to a certain tortuosity mainly owing to the heterogeneity of the media themselves. Consequently, to yield any significant result from equation (17) it is necessary to smooth over the tortuosity of the observed $w-\eta$ curve, which often brings in uncertainty and arbitrariness. The error of this method was estimated qualitatively by Bruce and Klute to be 200–500% [9].

Following this method, we measured the mass diffusivity of sand at room temperature. The traditional gravimetric sampling was used to obtain the moisture content profiles as shown in Fig. 1. Our apparatus is shown in Fig. 3. A testing column (1) consisted of sections of glass tubing, 11 mm long and 35 mm i.d. Both sides of the section were polished

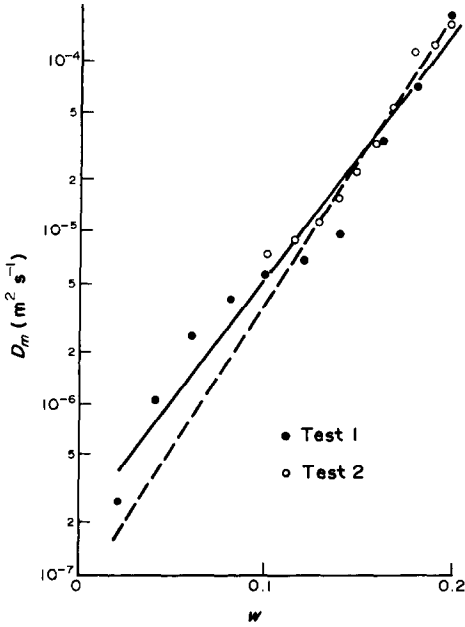


FIG. 4. The mass diffusivity of sand.

carefully so that they could be fitted tightly together. The clamping tube (2) was used to fix the glass tubing and to serve as an inlet for testing materials. A Mariotte-type burette (3) supplied water at a constant head and measured the amount of water supplied. The specimen was segregated with the piping by brass meshes and a sheet of filter paper at the wetting end, where an exhaust valve (4) was installed for air exhaustion. The column (1) was connected first with a funnel (5) through a tee-valve (6) to replace air in the piping. Then, the exhaust valve (4) was closed and the tee-valve turned to the burette (3) while a stopwatch was started for counting the time duration. When the absorption had proceeded for the desired time, the water supply was cut off, the specimen was separated quickly and the moisture content in each section was determined by the gravimetric procedure.

Two moisture content profiles so obtained with different initial moisture contents were used to evaluate the mass diffusivity of the specimen. The data are plotted in Fig. 4, and the solid line in the figure is the regressive exponential function from these data, which yields

$$D_m = 1.94 \times 10^{-7} \exp(33.3w) \text{ [m}^2 \text{ s}^{-1}\text{]}.$$

It is clear that

$$I = \Sigma \rho_a h(w - w_i) \tag{18}$$

where h is the length of each section. From equation (16) we have

$$\bar{D}_m = \frac{\pi}{4\tau} \left[\frac{\Sigma h(w - w_i)}{w_o - w_i} \right]^2. \tag{19}$$

The same experimental data were also used to evaluate the mean mass diffusivity, and the results are listed

in Table 1. The dashed line in Fig. 4 was obtained according to these data listed in Table 1 by the relationship between \bar{D}_m and D_m as discussed below.

5. RELATIONSHIP BETWEEN \bar{D}_m AND D_m

For an arbitrary diffusivity $D_m = D_m(w)$ the relationship between \bar{D}_m and D_m cannot be formulated in a concise expression, and \bar{D}_m has to be calculated from the cumulative absorption computed by a numerical method. We discuss below the absorption in porous media with the assumed exponential expression of the mass diffusivity, equation (12). Meanwhile Brutsaert's approximate solution, equation (11), is also adopted in the following discussion.

Rewriting equations (16) and (12) with the normalized moisture content, $W = (w - w_i)/(w_o - w_i)$, we have

$$\bar{D}_m = \frac{\pi}{4} \left[\int_0^1 \eta dW \right]^2 \tag{20}$$

and

$$\bar{D}_m = D_{m,o} \exp[C(W - 1)] \tag{21}$$

where

$$C = c(w_o - w_i).$$

Integrating equation (6) and using equation (11) yields

$$\begin{aligned} \int_0^1 \eta dW &= -2 \left(D_m \frac{dW}{d\eta} \right)_{\eta=0} \\ &= 2 \left[\int_0^1 W^{1/2} D_m(W) dW \right]^{1/2}. \end{aligned} \tag{22}$$

Substituting equation (21) into equation (22) and integrating, we obtain

$$\bar{D}_m = \frac{\pi}{4} D_{m,o} \frac{2C - 1}{C^2} [1 + F(C)] \tag{23}$$

where

$$F(C) = M(-0.5, 0.5, C) \exp(-C)/(2C - 1) \tag{24}$$

$M(a, b, z)$ is referred to the confluent geometric function [11], and here it turns out to be a simple series

$$M(-0.5, 0.5, C) = - \sum_{n=0}^{\infty} \frac{C^n}{(2n - 1)n!} \tag{25}$$

Functions $M(-0.5, 0.5, C)$ and $F(C)$ are plotted in Fig. 5. It shows that $|F(C)| < 0.02$ as $C \geq 4.8$, therefore equation (23) may be reduced for larger C values to

$$\frac{\bar{D}_m}{D_{m,o}} = \frac{(2C - 1)}{4C^2}. \tag{26}$$

Equation (23) indicates that $\bar{D}_m/D_{m,o}$ is a function of $C = c(w_o - w_i)$ only. Having known the mass diffusivity parameters $D_{m,o}$, c and the absorption con-

Table 1. Determination of the mean mass diffusivity of sand

No.	w_o	w_i	$h(w - w_o)$ [cm]	τ [s]	\bar{D}_m [m ² s ⁻¹]
1	0.200	0.002	2.625	370	3.73×10^{-5}
2	0.200	0.097	1.139	145	6.62×10^{-5}

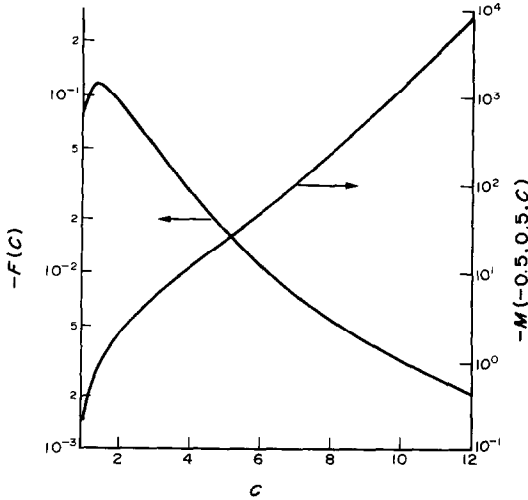


FIG. 5. Functions $M(-0.5, 0.5, C)$ and $F(C)$.

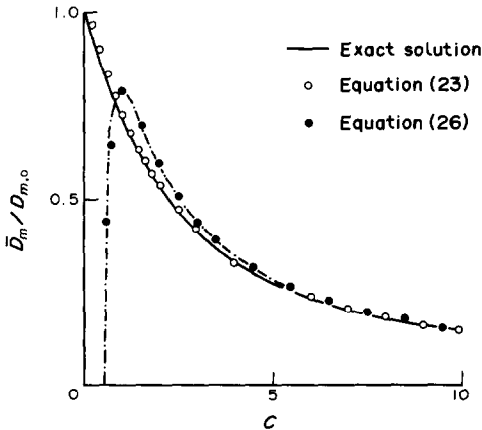


FIG. 6. The mean mass diffusivity calculated from the exact and approximate solutions.

ditions w_o and w_i , one will be able to evaluate \bar{D}_m and the cumulative absorption easily.

In order to examine the accuracy of equations (23) and (26), which have been derived from the approximate solution, equation (11), the exact \bar{D}_m is also computed by Philip's numerical solution [5] and is plotted in Fig. 6 together with the results obtained from equations (23) and (26). It is seen from the comparison that the accuracy of equation (23) will be up to within 0.3% as $C > 2$, while equation (26) is less accurate but still useful for engineering applications at larger C .

To discuss the effect of the initial moisture content on the mean mass diffusivity, a certain initial moisture content w_i^* is chosen as a reference, and a normalized

initial moisture content $\phi = (w_i - w_i^*) / (w_o - w_i^*)$ is introduced to represent different cases, while the moisture content on the boundary, w_o , is kept unchanged. Let $R = \bar{D}_m / \bar{D}_m^*$, where \bar{D}_m^* is the mean mass diffusivity of the reference case, then we get from equation (23)

$$R = \frac{1}{(1 - \phi)^2} \left[1 - \frac{2C^*\phi}{2C^* - 1} \right] \frac{1 + F[C^*(1 - \phi)]}{1 + F[C^*]} \quad (27)$$

where $C^* = c(w_o - w_i^*)$ and then $C = C^*(1 - \phi)$. It can also be reduced roughly to

$$R = \frac{1}{(1 - \phi)^2} \left(1 - \frac{2C^*\phi}{2C^* - 1} \right). \quad (28)$$

Equation (27) indicates that $R = \bar{D}_m / \bar{D}_m^*$ is a function of C^* and ϕ , and independent of $D_{m,o}$.

According to the established relation $R = R(C^*, \phi)$, it is possible to evaluate C^* from R and ϕ determined from two absorption experiments with different initial moisture contents. In the computation, as estimation of C^* was first given by equation (28), then Philip's numerical method [5] was used to search for the exact C^* . Finally c and $D_{m,o}$ can be figured out from the obtained C^* and equation (23), respectively. As a working example, the dashed line in Fig. 4 was obtained in this way.

6. ERROR ANALYSIS

In order to verify the feasibility of the method proposed and to design appropriate experimental parameters, error transmission should be considered. Equation (27) is preferred here to the numerical method for its advantages of being derivable and fine accuracy.

The relative errors in C^* resulting from those in R and ϕ can be written respectively as

$$\left(\frac{\Delta C^*}{C^*} \right)_R = \frac{R}{C^*} \left(\frac{\partial C^*}{\partial R} \right)_\phi \frac{\Delta R}{R} \quad (29)$$

$$\left(\frac{\Delta C^*}{C^*} \right)_\phi = \frac{\phi}{C^*} \left(\frac{\partial C^*}{\partial \phi} \right)_R \frac{\Delta \phi}{\phi}. \quad (30)$$

According to the basic rules of differential calculus, we have

$$\left(\frac{\partial C^*}{\partial R} \right)_\phi = 1 / \left(\frac{\partial R}{\partial C^*} \right)_\phi \quad (31)$$

$$\left(\frac{\partial C^*}{\partial \phi} \right)_R = - \left(\frac{\partial R}{\partial \phi} \right)_{C^*} / \left(\frac{\partial R}{\partial C^*} \right)_\phi \quad (32)$$

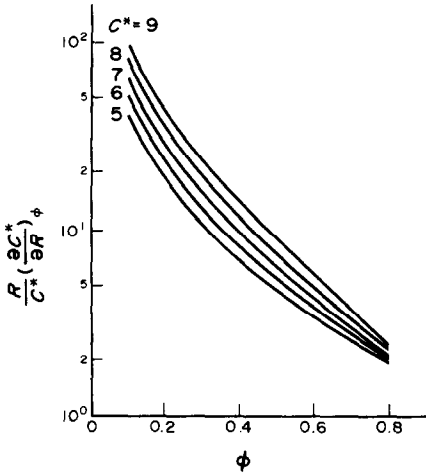


FIG. 7. Transitive factor of error from R to C^* .

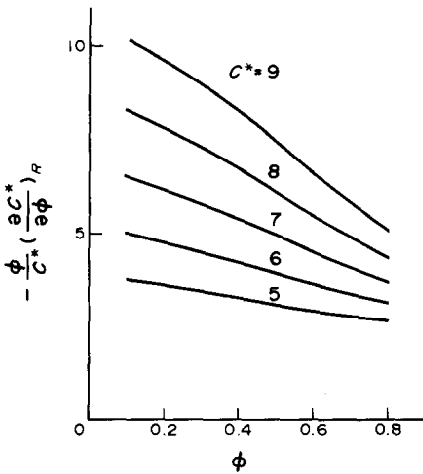


FIG. 8. Transitive factor of error from ϕ to C^* .

$$\begin{aligned} \left(\frac{\partial R}{C^*}\right)_\phi &= \left[\frac{\partial(\bar{D}_m/\bar{D}_m^*)}{\partial C^*}\right]_\phi \\ &= \left[\bar{D}_m^*(1-\phi) \frac{d\bar{D}_m}{dC}\Big|_{C=C} - D_m \frac{d\bar{D}_m}{dC}\Big|_{C=C^*}\right] / (\bar{D}_m^*)^2 \end{aligned} \quad (33)$$

$$\left(\frac{\partial R}{\partial \phi}\right)_{C^*} = -\frac{C^*}{\bar{D}_m^*} \frac{d\bar{D}_m}{dC}\Big|_{C=C} \quad (34)$$

From equations (23) and (24), we obtain

$$\begin{aligned} \frac{d(\bar{D}_m/D_{m,o})}{dC} &= \frac{\pi}{2} \left[\frac{1-C}{C^3} \right. \\ &\quad \left. + \frac{C \cdot M'(-0.5, 0.5, C) - (2+C) \cdot M(-0.5, 0.5, C)}{2C^3 \exp(C)} \right] \end{aligned} \quad (35)$$

The transitive factors of error, $(R/C^*)(\partial C^*/\partial R)_\phi$ and $(\phi/C^*)(\partial C^*/\partial \phi)_R$, are plotted in Figs. 7 and 8, respectively. The figures show that a greater ϕ , or a

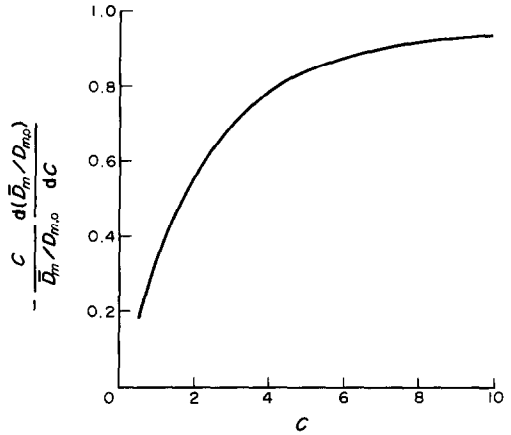


FIG. 9. Transitive factor of error from c to $D_{m,o}$.

higher w_i , and a smaller C^* , or smaller $(w_o - w_i)$, will be favourable for reducing the error in evaluating C^* .

The relative error in $D_{m,o}$ resulting from that in c is

$$\begin{aligned} \left(\frac{\Delta D_{m,o}}{D_{m,o}}\right)_C &= \frac{C}{D_{m,o}} \left(\frac{\partial D_{m,o}}{\partial C}\right)_{D_m} \frac{\Delta C}{C} \\ &= -\frac{CD_{m,o}}{\bar{D}_m} \frac{d(\bar{D}_m/D_{m,o})}{dC} \frac{\Delta C}{C} \end{aligned} \quad (36)$$

This transitive factor of error is shown in Fig. 9. It is clear that a smaller C is also favourable to obtain a more reliable $D_{m,o}$.

7. CONCLUDING REMARKS

The dependence of the mean mass diffusivity for isothermal absorption in porous media upon the initial moisture content and its relationship with the actual mass diffusivity have been derived. Both Bruce and Klute's method [9] and a proposed method were used to determine the mass diffusivity of wet sand as a function of moisture content. The proposed method considers only the overall effect of absorption, and so the measurement is supposed more reliable. The obtained mean mass diffusivity can be used to evaluate the cumulative water absorption and to determine further the exponential expression of the mass diffusivity.

The error transmission in determining the exponential parameters, $D_{m,o}$ and c , of the mass diffusivity is discussed in detail. However, it is still hard to estimate quantitatively the accuracy in the results because the errors of respective tests depend to a considerable extent upon homogeneity and reproducibility of the specimens.

The results obtained by the two different methods agree with each other reasonably, thus verifying the feasibility of these methods.

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REFERENCES

1. A. V. Luikov, *Heat and Mass Transfer in Capillary-porous Bodies*. Pergamon Press, Oxford (1966).
2. B. X. Wang and Z. H. Fang, Heat and mass transfer in wet porous media and a method proposed for determination of the moisture transport properties, *Heat Technol.* **2**(1), 29–42 (1984).
3. J. R. Philip, Theory of infiltration, *Adv. Hydrosci.* **5**, 215–296 (1969).
4. D. Swartzendruber, The flow of water in unsaturated soils. In *Flow through Porous Media* (Edited by R. J. M. de Weist), pp. 215–292. Academic Press, New York (1969).
5. J. R. Philip, Numerical solution of equations of the diffusion type with diffusivity concentration dependent, *Trans. Faraday Soc.* **51**, 885–892 (1955).
6. J. Y. Parlange, On solving the flow equation in unsaturated soil by optimization : horizontal infiltration, *Soil Sci. Soc. Am. Proc.* **39**, 415–418 (1975).
7. W. Brutsaert, The concise formulation of diffusive sorption of water in a dry soil, *Water Resour. Res.* **12**, 1118–1124 (1976).
8. W. R. Gardner and M. S. Mayhugh, Solutions and tests of the diffusion equation for the movement of water in soil, *Soil Sci. Soc. Am. Proc.* **22**, 197–201 (1958).
9. R. R. Bruce and A. Klute, The measurement of soil moisture diffusivity, *Soil Sci. Soc. Am. Proc.* **20**, 458–462 (1956).
10. R. J. Gummerson, C. Hall and W. D. Hoff, Unsaturated water flow within porous materials observed by NMR imaging, *Nature* **281**, 56–57 (1979).
11. M. Abramowitz and I. A. Stegun (Editors), *Handbook of Mathematical Functions*, Applied Mathematics Series, Vol. 55, pp. 504–535. National Bureau of Standards, Washington, D.C. (1964).

ABSORPTION D'EAU ET MESURE DE LA DIFFUSIVITE DE MASSE DANS LES MILIEUX POREUX

Résumé—On résume les études d'absorption d'eau dans les milieux poreux. La diffusivité moyenne de masse pour la migration d'humidité est une propriété constante reliée à la diffusivité de masse réelle et on présente sa dépendance vis-à-vis du contenu initial d'humidité. On propose une méthode pour déterminer les paramètres dans l'expression exponentielle de la diffusivité de masse en fonction de la diffusivité moyenne mesurée de masse. On discute l'erreur de transmission correspondante et les résultats expérimentaux avec du sable sec sont comparés avec les courbes calculées.

WASSERAUFNAHME UND MESSUNG DES DIFFUSIONSVERMÖGENS IN PORÖSEN MEDIEN

Zusammenfassung—Es wird eine kurze Übersicht über Untersuchungen über die Wasseraufnahme in porösen Medien gegeben. Das mittlere Diffusionsvermögen für die Feuchtigkeitswanderung im Sinne einer konstanten Eigenschaft steht in Beziehung zum momentanen Diffusionsvermögen. Seine Abhängigkeit vom anfänglichen Feuchtigkeitsgehalt wird dargestellt. Es wird eine Methode für die Ermittlung der Parameter in dem exponentiellen Ausdruck für das Diffusionsvermögen vorgeschlagen. Die entsprechende Fehlerfortpflanzung wird diskutiert. Experimentelle Ergebnisse für nassen Sand werden mit den berechneten Kurven verglichen.

ПОГЛОЩЕНИЕ ВОДЫ И ИЗМЕРЕНИЕ МАССОПРОВОДНОСТИ В ПОРИСТЫХ СРЕДАХ

Аннотация—Дан краткий обзор исследований по поглощению воды в пористых средах. Приводится связь средней массопроводности для миграции влаги при постоянных свойствах с истинной массопроводностью, представлена зависимость от начального содержания влаги. Предложен метод для определения параметров в экспоненциальном выражении для массопроводности, соответствующем измеренному среднему значению массопроводности. Обсуждается погрешность метода; экспериментальные результаты, полученные для мокрого песка, сравниваются с расчетными кривыми.